About the Towers of Hanoi puzzle – mostly for teachers

This is a very flexible puzzle which can lead to mathematical analysis, if you want to go that far.

Primary children usually enjoy using 2,3, and 4 rings. Older children enjoy the challenge of getting up to 5 rings. Once students understand exponential notation, it is possible to do the full mathematical analysis, which I've done with everyone from grade 6 to university students.

I often use this game on the first day of my Math for Teachers class, after which students tell me that it makes them think that the course "might not be so bad after all". A colleague uses it in her discrete math course for computing students, and I've used it in a third-year number theory course.

Besides all that educational value, the Towers of Hanoi puzzle is enough fun to be a big draw at Math Mania nights, at my university's summer science camps for grade 4-6, and at our afterschool science programs for elementary students.

Playing the game in class:
Students in a classroom setting seem to get the most out of it when they work in pairs, so you'll want to make many small copies: I started with cardboard, paper clips and coloured foam. My father then made a sturdier set of small versions out of wood. These have been borrowed frequently!

I explain the rules, demonstrating with the big set. After 10-15 minutes, we collect results on the board. Most pairs figure out the minimum number of moves for 1-5 rings and then predict correctly the values for 6 and 7 but have a hard time with 25. We compare the patterns the students have discovered. Some find a pattern based on differences (they increase by 4, 8, 16, etc). Some notice that each value for n is twice the previous one, plus one; I demonstrate how this arises physically*. Very occasionally students will find the pattern that will allow them to predict the number of moves for 25 rings, then n. If so, I have them explain it; if not, we work it out together.

This is what the complete table should look like:

# of rings	1	2	3	4	5	6	7	25	n
minimum # of moves	1	3	7	15	31	63	127	$2^{25}-1$	2^n-1

*ex: to move a stack of 5, you need to move the top 4 first; we already know that takes 15 moves; then move the 5^{th} ring to reach 15+1 moves; then move the 4 again, which takes us to 15+1+15.

Taking it further:

- Find out who invented this game, when, and what the story was to go with it.
- Answer the original question - how long to move 64 rings if each move takes one second?
- Look at some variations: (a) two or three colours, with no two of same colour being allowed to touch, (b) what happens with four spikes? (this leads to a discussion of how easy it can be to ask extremely difficult questions!)
- Make up your own variations.

Comments:

The class activity demonstrates that different kinds of questions can be associated with the same situation. We also talk about organising data, assigning variables, finding different but equally correct patterns, figuring out how to write down patterns, exponential notation, and generalising results in different ways. We discuss how the mystifying $2^0 = 1$ arises.