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Recreational mathematics and outreach are two threads that have woven their ways into the Education Notes, particularly the past year. This article by Susan Milner builds upon these themes. Many people are engaged in broadening the lens for mathematical viewing in various forms. Contributions from individuals or teams of people are welcomed. Send along articles and ideas to Jennifer Hyndman (hyndman@unbc.ca) or John Grant McLoughlin (johngm@unb.ca).

Puzzles in my life

Susan Milner, *University of the Fraser Valley (susan.milner@ufv.ca)*

For years puzzles, physical and later on-line, were a private addiction, until I discovered that the physical puzzles made great icebreakers at our departmental and social events. Of course, one would expect math-type people to enjoy puzzles, but at these events even those less mathematically inclined seemed engaged.

One year the University of the Fraser Valley (UFV) asked interested departments to host tables as part of student orientation in late August. Well, if math-type people enjoy physical puzzles, why not have a few at our table? So to our bookmarks and handouts, I added brightly coloured two-and three-dimensional geometric puzzles, wire tavern puzzles, Towers of Hanoi, and some eye-catching commercial logic games such as *Rush Hour*. We even had cheap little wire puzzles to give away to people who seemed truly fascinated. It was rather a success.

Goodness knows if the new students absorbed much of what they were supposed to, but they were definitely keen to play. ("Yes, mathematicians play all day!") Our table was mobbed for the whole time. Wistfully, faculty came over from other tables; we gave them something to play with while they were waiting for people to talk to. The next year, the invitation to the orientation event mentioned bringing "small puzzles or other fun things".

Not all the students surrounding our table at Orientation plan to take university math courses. I have had good discussions with cook-training students and fine arts students, for example. They are just as fascinated by various puzzles as are the future physicists, computer scientists, and engineers, who seem on the face of it to be a more likely audience. "I didn't like [insert your favourite high school math topic] but I love doing puzzles" is a frequent comment, as is, "What does this have to do with math?"

Ah, now we are talking. How is solving puzzles connected to doing mathematics?

Why say "of course, one would expect math-type people to enjoy puzzles?" What is involved beyond the obvious – the pleasure of getting the right answer?

It is one thing to say that doing puzzles is good for the brain, that learning different games creates new connections, that it might help to ward off stultification and even dementia. I am perfectly happy to justify it that way. Learning a new game makes me feel awake. Mastering a game, playing it at increasingly difficult levels, makes the rest of the world go away for a bit – it's a wonderful way to clear the mind. These things are true of many activities and are enough in themselves to commend puzzles to our attention. No doubt these reasons are why puzzles have become very popular recently. This year I gave a six-week course called Brain Games as part of UFV's Elder College, to a small but keen group of 50- to 80-year-olds. They were there not because they particularly liked math – some actively disliked what they recalled of it – but because they wanted to exercise their brains. At the end of the six weeks, they were keen to have another course next winter, so they clearly found the puzzles stimulating and worthwhile. Beyond this, though, I am increasingly interested in the direct connections between puzzles and mathematical thinking. I would like to know more.

In 2006 I started teaching *Math for Elementary Teachers*, and as anyone who teaches such courses knows, students vary widely in age, experience, and comfort level with mathematical ideas. Some of my students could write the weekly quiz in five minutes while others took fifteen. I hate having people just staring at the ceiling, waiting for others to finish! And so, wanting to set a good example for these future teachers about keeping everyone involved at all times, I started handing out a puzzle with each quiz, a different type of puzzle every week. Students could start on the puzzle if they finished the quiz early or take it home to play with.

It started out purely for amusement, but soon I started to hear interesting things:

- Some students enjoyed the puzzles and wanted more so they could share them with their own children.
- More than one person related how they were now using puzzles as encouragement – or bribes – for children they were working with. One said, "I'm working with a girl who just hates numbers. She loved your Sudoku with the letters M A T H, so I made one up for her with her name and said she could do it after she finished her math homework. It worked! She wants more."
- Someone told the class, amazed, "We did the Towers of Hanoi at the Y daycare; the kids didn't want to leave until they'd figured it out. They were so proud that they were solving a difficult problem – they made their parents wait for them."
- Some students didn't understand a puzzle at first but were thrilled to tell me when they did figure it out.

- Some of these students were delighted to discover that they could learn to solve a seemingly difficult puzzle by finding manipulatives to model it.
- Some didn't understand a puzzle until a friend, spouse, or their own child worked through it with them; they told me how they had really enjoyed the experience of figuring things out with that other person.

They all were keen to learn new puzzles. It wasn't uncommon for someone to say that they now had to spend time playing a particular puzzle online every day. *Oops, another addict.*

Of course, on being given a task with no follow-up, some students just ignore the whole thing. To encourage these students to make an effort, I began putting a short puzzle on our tests. This quickly developed from bonus question to required: "You should be able to do one of these three types of puzzles at a level appropriate for elementary school students."

	6		22	
	4	**25**	24	
	9		1	
	12		16	
	14	15	18	

Hidato

Smaller versions of this puzzle appeal to kindergarten and grade 1 students, while older students appreciate larger and more complex versions.

Make a chain from 1 to 25, connecting the squares vertically, horizontally, or diagonally.

For printable puzzles meant for primary grades, see the MathinEnglish site. Hidato, the official website, has much larger on-line puzzles.

Why make puzzles a required part of a math course?

- Students were paying close attention to instructions and asking questions if they didn't understand. *First step to solving a problem! Words mean something.*
- Students who had shown a serious lack of confidence were trying harder to solve the puzzles than they tried to solve classroom problems. *Tenacity is huge.*
- Many were enjoying something mathematical for the first time that they could remember. They cared enough to want to solve puzzles but they didn't get so anxious that they couldn't think straight. *Major shift in attitude: I want this group to see that change is possible. Anyone teaching math to children needs to know this at a visceral level.*
- Students were explaining their reasoning to each other. Not that some students didn't already do this with homework problems, but it wasn't always the usually-confident people doing the explaining. Students who'd told me "I can't do math" displayed excellent logic. *By any other name: modus ponens, modus tolens, proof by contradiction, law of the excluded middle, the difference between "this might be true" and "this must be true", long chains of reasoning, awareness of branch points in an argument.… And not only "math brains" can play.*
- Spatial reasoning, usually two-dimensional, is necessary for many of these puzzles. One that requires three-dimensional visualisation is *Towers/Skyscrapers/Utopia* – it is best to start with physical towers, colour-coded if possible, but eventually one can use just a pencil and paper. *Thinking in pictures is a huge help with many mathematical problems. We know this but it comes as a surprise to many students.*
- Different students showed strengths with different puzzles. *No stigma is attached to liking one type of puzzle over another. Mathematical thinking comes in forms that are different yet connected in fundamental ways.*

As long as I give them a choice of puzzles to solve in testing situations, my students enjoy showing off their puzzling skills. When a test contains an element that students feel is both playful and valuable, something good is happening.

	3		2	1	
4					
2					2
					4
2					2
	2	1	3	2	

Towers *(or Skyscrapers, or Utopia)*

The clues describe how many buildings are visible from that position. Each of the four heights must appear in each row and each column.

Brainbashers and Simon Tatham's Portable Puzzles have good on-line versions.

While the confidence and skills people develop in response to a given situation may not transfer directly to other situations, it seems likely that there is at least some carry-over. Short discussions about overall problem-solving skills in connection with puzzles do seem to encourage my students to approach the dreaded "word problems" with more confidence and consequently obtain better results. Some reflective, mature students have said specifically that learning how to solve the puzzles helped them handle the course material, to the point where they surprised themselves by doing very well in a course they had dreaded. More importantly, they were now excited and more confident about teaching mathematics to children.

These observations all come out of my experience with students in that one course, but they have been borne out again and again over the past six years of teaching the course. Maybe puzzles and games can play a small part in changing the Canadian pattern of having the math-averse and math-fearful pass on their issues to the next generation.

More venues for spreading the love

For 16 years, UFV has hosted local participants in the *BC Secondary School Math Contest* for grades 8-12. The afternoon of the contest is spent entertaining the contestants with various types of mathematical activities. Our ultimate goal is to ensure that the students have fun and see their peers also enjoying themselves doing something that is probably not at all common in their usual environments. I've had the great pleasure of sharing a number of my favourite puzzles and games with these students and their teachers, along with our own volunteers. This is admittedly a self-selected audience, but one that certainly enjoys learning new games. We get to try out some of the more challenging ones and sometimes we even think a bit about the underlying mathematics.

In 2009 my department started offering *Math Mania* events, with the support of the Pacific Institute for the Mathematical Sciences (PIMS). Four times a year we take a whole lot of games and puzzles, some home-made and some available commercially, along with 18-25 volunteers into a local elementary school, set up tables around the gym, and let the children, parents, and teachers wander around at will. It is fascinating to watch the parents, many of whom are ill at ease around school in general and/or mathematics in particular, sidle into the gym, eye the games, and gradually get drawn into the action. They often stand behind their children and give advice, until we can nudge them into trying something themselves. "This is mathematics?", some ask. We might then try to explain how it is related to mathematical thought. Mostly we watch their eyes light up at the sight of the delight of their own children in solving complicated problems. I don't know who has the most fun - the kids, their parents, or our volunteers. Interestingly, many of our volunteers come back year after year because they enjoy the experience so much. Volunteers include students, faculty, retired faculty, as well as children and spouses of students and faculty. And not all the faculty involved are mathematicians – some of our regulars come from biology, physics, computing, and philosophy. *If everyone can find mathematical/logical puzzles and games that they enjoy and are good at, then everyone can engage in mathematical thinking. It becomes a matter of degree rather than an abyss between those who can and those who can't.*

Over the past several years, I've enjoyed sharing games with the students or the teachers at Simon Fraser University's summer *Math Camps*. I've also learned new games from the people there. These audiences are also self-selected, of course; the point here isn't "*you too can do mathematics"* but rather *"look how you can push yourself to think mathematically in new contexts."*

Recently, much interest has been expressed around puzzles and games at my two favourite conferences, *Changing the Culture* and *Sharing Mathematics*, each held annually in British Columbia. I've learned a great deal and I've had a chance to give a few workshops about my favourite games, too. I find it particularly exciting when there are primary or secondary teachers in the audience, so that we can all think about how puzzles and games can be used at different levels. I vividly remember one secondary teacher saying that his formerly tardy class now showed up on time and ready to go because he started every day with 10 minutes of puzzles. Another teacher said that for her grade 11 & 12 students, "puzzles are a better bribe than candy".

This year has been the best so far in my world of puzzling – I've been on sabbatical and able to devote myself almost entirely to puzzles and games. I had hoped to visit up to a dozen K-12 classrooms, try out some of my games with the students, then spend the rest of the time thinking, studying what other people are doing, and creating an online resource for K-12 teachers.

The year has, however, been considerably more active than contemplative. I did not have to figure out ways to convince teachers and principals that this is worthwhile – many are already very interested in using math puzzles and games in the classroom. With help from Science World's *Scientists & Innovators in Schools* (SIS) program, I ended up visiting 191 classrooms during 55 days of intense hands-on activities, involving almost 4200 students. *Whew!*

	3			2	2		3	
5						3		
								8
5	4		4	9			2	
							2	5
	4							
		8		3	3			
							6	

Rectangles *(Shikaku)*

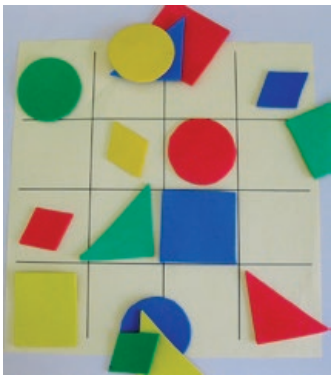
This puzzle combines factoring numbers with spatial reasoning.

The goal is to cover the grid with non-overlapping rectangles, each of which contains exactly one numeral. The numeral specifies how many of the small boxes are in each rectangle.

Brainbashers and Simon Tatham's Portable Puzzles have good on-line versions.

I learned a great deal about what appeals to students of various ages, what they are capable of in terms of logical argument, and how to

structure games to work well in a classroom setting. The students generally responded very well to the games. Their teachers were pleased and often surprised at how much excitement, higher-level thinking, and stick-to-it-iveness their students displayed. They liked the fact that the games are graduated in difficulty, which allows for differentiated learning, and often incorporate brightly coloured manipulatives, which increases their appeal to a variety of students. Many teachers told me something along the lines of "I knew that Kim would love these games, but I had no idea that Alex would be so good at them", or "That student hasn't asked one question in the three years I've had her; she actually asked you something!" One child with cerebral palsy, who struggles with letters and numbers alike, amazed his teacher by being among the fastest in the class to create a 4x4 Euler square using coloured shapes.



Latin and Euler squares

There are several variations one can work through. This one avoids repetition of either shape or colour in rows, columns and main diagonals.

Wherever possible I chatted with the teachers about the curriculum goals that can be addressed by such games. These include: encouraging curiousity, encouraging a willingness to try various approaches, developing logical thinking using sequential and spatial reasoning, helping students learn to develop their own strategies for solving problems, and developing tenacity. One of the stated goals of BC's K-12 curriculum is to provide "positive learning experiences [that] build self-confidence and develop attitudes that value learning mathematics." Judging by the reactions in the classrooms I visited, these games provide positive learning experiences. Common responses included "Best math class ever!", "When are you coming back?", and "Can I take some of the harder ones home with me?" Some of this is no doubt due to the sheer novelty of it all, but I am hopeful that the effects can be longer-lasting if students are exposed to games and puzzles on an ongoing basis.

Not only were teachers and principals interested in classroom visits, I was invited to give professional development workshops, including one during the middle of the summer holidays. PIMS was very supportive, enabling me to give ten workshops around the province. Overall, including both classroom visits and professional development workshops, about 375 K-12 teachers this year have joined me in the joy of puzzling. Many of them were very keen to start using puzzles and games in their math classrooms, or to use more of them than they currently do, and nearly every one of these teachers signed up for access to the puzzles and games in my Dropbox. I think the interest is genuine, especially among those who have seen their students enjoying the puzzles. I would like to think that this will encourage more students to get engaged in mathematical thinking at a deeper level than they might otherwise have done, which will in turn open up a world of ideas and career options that might otherwise have been hidden from them.

Embracing the opportunity

This seems to be a very good time for those of us who enjoy mathematical/logical puzzles to spread the love. There are many related websites meant for K-12 teachers and students, although it has to be said that some of them are unutterably dreary, with "games" that are just disguised worksheets, so one has to check them out carefully. Real puzzles exist in the world outside the classroom; we might modify them a bit to bring them to the appropriate level for a given grade, but they have to be entertaining enough for people to do them when they are not being coerced. Kids can tell the difference!

I am sure that many readers of the *CMS Notes* have their own collection of favourites that work in their classrooms or the classrooms they visit. Maybe we could start collecting the favourites in a super-resource. Who better for the task than puzzle-loving mathematicians?

Domino puzzles

A good example of a useful modification: using only partial sets of dominoes makes for more accessible – and popular – puzzles.

Susan Milner is a long-time member of the Mathematics & Statistics Department at University of the Fraser Valley in British Columbia. She welcomes your comments about the above essay and also about the use of mathematical/logical puzzles and games in the classroom. If you would like access to her Dropbox of puzzles and games, please e-mail her at susan.milner@ufv.ca.